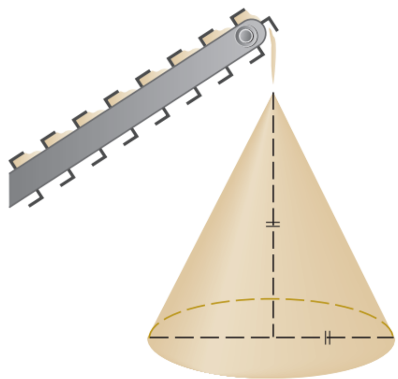
Exercise 29

Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?



Solution

Start with the formula for the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h \tag{1}$$

Since we want to find dh/dt when $h = 10$, the aim is to eliminate r in favor of h. The height and diameter are always equal, so

$$r = \frac{h}{2}.$$

As a result, equation (1) becomes

$$\begin{aligned} V &= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h \\ &= \frac{\pi}{3}\left(\frac{h^2}{4}\right) h \\ &= \frac{\pi}{12}h^3. \end{aligned}$$

Take the derivative of both sides with respect to t by using the chain rule.

$$\begin{aligned} \frac{d}{dt}(V) &= \frac{d}{dt}\left(\frac{\pi}{12}h^3\right) \\ \frac{dV}{dt} &= \frac{\pi}{12}(3h^2)\cdot\frac{dh}{dt} \\ 30 &= \frac{\pi}{4}h^2\frac{dh}{dt} \end{aligned}$$

Solve for dh/dt.

$$\frac{dh}{dt} = \frac{120}{\pi h^2}$$

Therefore, the rate that the height of the pile is increasing when the pile is 10 feet high is

$$\left.\frac{dh}{dt}\right|_{h=10} = \frac{120}{\pi(10)^2} = \frac{6}{5\pi} \approx 0.381972\ \frac{\text{ft}}{\text{min}}.$$